The Role of GammaF (γ_f) in Two-way Slab Punching Shear Calculations (ACI 318-14)

Objective

Determine the adequacy of two-way (punching) shear strength around the exterior and interior columns in a typical two-way flat plate concrete floor system per ACI 318-14. Perform the unbalanced moment transfer calculations at slab-column connections. Determine the possible utilization of increased γ_f (GammaF) procedure in order to minimize or eliminate entirely the need for shear reinforcement.

Codes

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

References

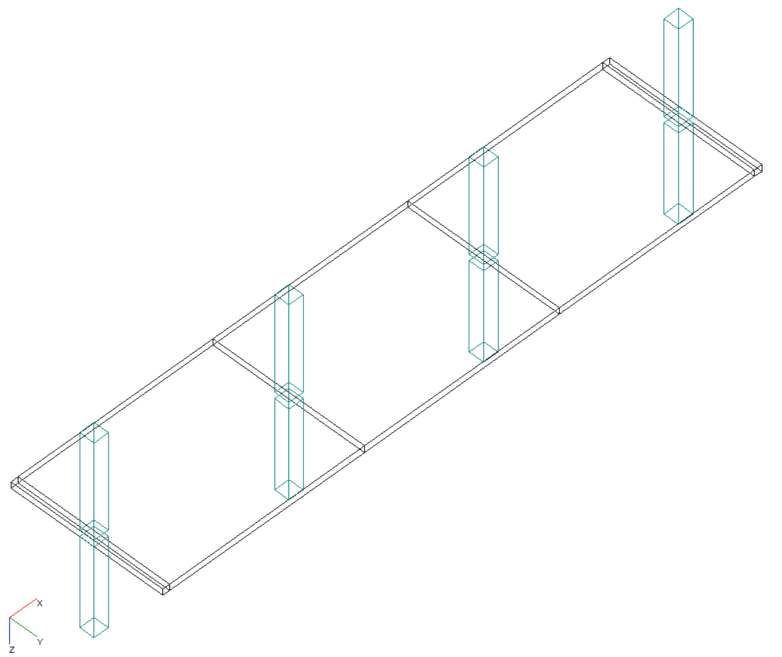
[1] Notes on ACI 318-11 Building Code Requirements for Structural Concrete with Design Applications, Edited by Mahmoud E. Kamara and Lawrence C. Novak, Portland Cement Association, 2013

[2] Wight J.K., Reinforced Concrete, Mechanics and Design, Seventh Edition, Pearson Education, Inc., 2016

Contents

Two-way Slab Model – Geometry & Design Data

The isometric and plan views of the two-way flat plate concrete slab below are generated from an analytical model using the spSlab Program.



Slab thickness, h = 7 in.

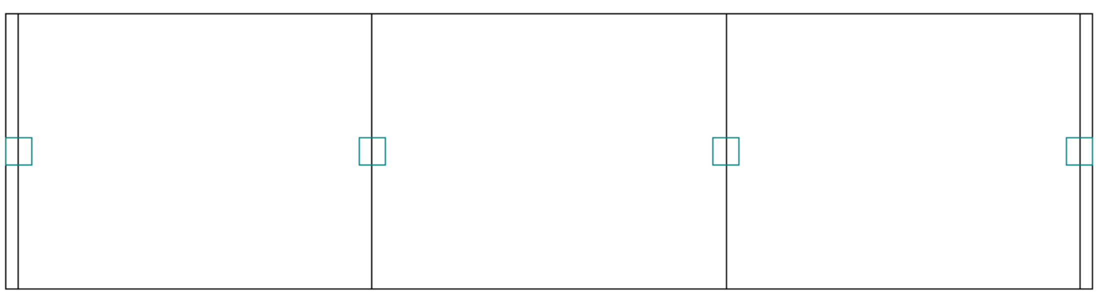
d = 5.75 in.

f_c' = 4,000 psi (for slabs)

f_c' = 6,000 psi (for columns)

f_y = 60,000 psi

Isometric view of two-way flat plate concrete slab



Exterior Columns
c_1 = 16 in. c_2 = 16 in.
Location: Edge
Span direction: "Perpendicular to the edge"

Two-way shear perimeter dimensions
$b_1 = c_1 + d/2 = 16 + 5.75/2 = 18.88$ in.
$b_2 = c_2 + d = 16 + 5.75 = 21.75$ in.
$b_0 = 2\,b_1 + b_2 = 59.5$ in.

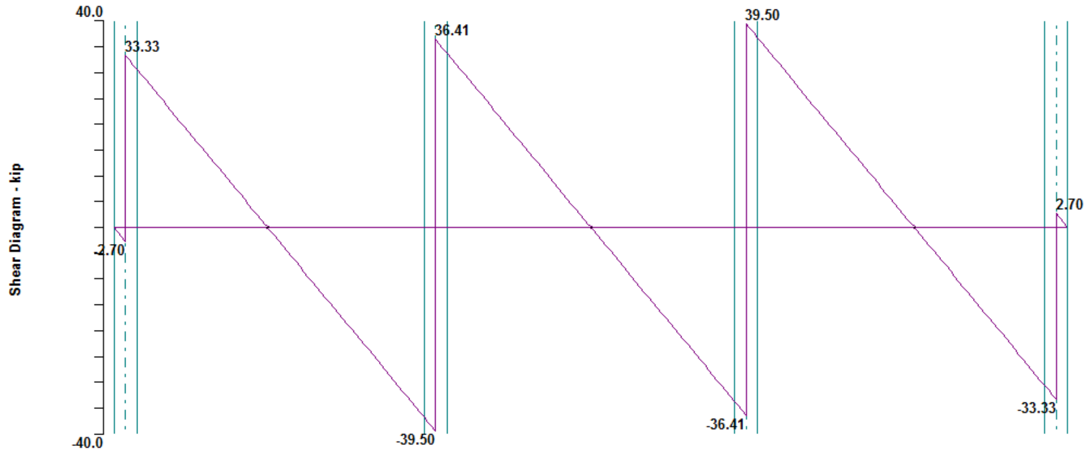
Interior Columns
c_1 = 16 in. c_2 = 16 in.
Location: Interior
Span direction: "Either direction"

Two-way shear perimeter dimensions
$b_1 = c_1 + d = 16 + 5.75 = 21.75$ in.
$b_2 = c_2 + d = 16 + 5.75 = 21.75$ in.
$b_0 = 2\,b_1 + 2\,b_2 = 87.0$ in.

Plan view of two-way flat plate concrete slab

Internal Force (Shear Force & Bending Moment) Diagrams

The shear force and bending moment diagrams below are produced by the spSlab Program.

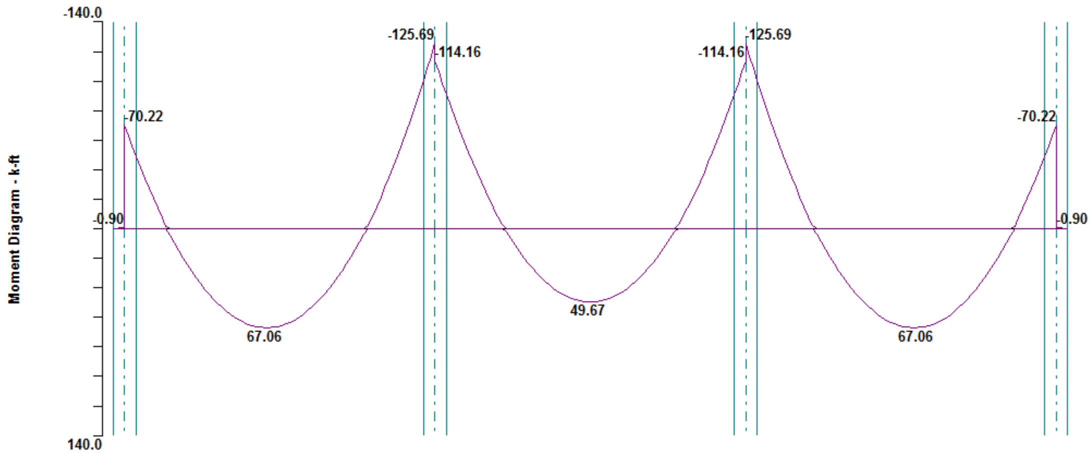


Shear Force Diagram from spSlab

From the shear force diagram, the factored shear force, $V_{u,c}$, that is resisted by the exterior and interior columns at a slab-column joint are:

At exterior supporting column: $V_{u,c}$ = [33.33 + 2.70] = 36.03 kip

At interior supporting column: $V_{u,c}$ = [36.41 + 39.50] = 75.91 kip



Bending Moment Diagram from spSlab

From the bending moment diagram, the factored slab moment, M_{sc}, that is resisted by the exterior and interior columns at a slab-column joint are:

At exterior supporting column: M_{sc} = [70.22 – 0.90] = 69.32 k-ft

At interior supporting column: M_{sc} = [125.69 – 114.16] = 11.52 k-ft

ACI 318-14, 8.4.2.3.2 states that the fraction of factored slab moment resisted by the column, $\gamma_f M_{sc}$, shall be assumed to be transferred by flexure, where γ_f shall be calculated by:

$$\gamma_f = \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}}$$

where

b_1 = Dimension of the critical section b_o measured in the direction of the span for which moments are determined

b_2 = Dimension of the critical section b_o measured in the direction perpendicular to b_1

ACI 318-14, 8.4.4.2.2 states that the fraction of factored slab moment resisted by the column, $\gamma_v M_{sc}$, shall be assumed to be transferred by eccentricity of shear, where γ_v shall be calculated by:

$$\gamma_v = 1 - \gamma_f$$

Compute the γ_f and γ_v values for exterior and interior column as follows:

Exterior Column: $$\gamma_f = \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{18.88}{21.75}}} = 0.617 \quad \text{and} \quad \gamma_v = 1 - 0.617 = 0.383$$

Interior Column: $$\gamma_f = \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{21.75}{21.75}}} = 0.600 \quad \text{and} \quad \gamma_v = 1 - 0.600 = 0.400$$

Per ACI 318-14, 8.4.2.3.3, all reinforcement resisting $\gamma_f M_{sc}$ shall be placed within the effective slab width, b_{slab}, which is between lines that are one and one-half the slab thickness, 1.5h, on each side of the column.

$$b_{slab} = c_2 + 2\times(1.5\times h) = 16 + 2\times(1.5\times 7) = 37 \text{ in.}$$

Two-way slabs without beams that are modeled by spSlab need to have the effective slab width, b_{slab}, located within the column strip width as defined by the Equivalent Frame Method (EFM). A detailed description of EFM is given in Chapter 2 of spSlab Manual.

Two-way (Punching) Shear Calculations

Two-way (punching) shear calculations are performed to ensure that the concrete slab design shear strength, ϕv_n , shall be greater than or equal to the factored shear stress, v_u .

$$\phi v_n \geq v_u$$

where strength reduction factor, ϕ, for shear equals to 0.75 per ACI 318

The combined two-way (punching) shear stress, v_u , is calculated as the summation of direct factored shear force alone and direct shear transfer resulting from the fraction of unbalanced factored moment:

$$v_u = \frac{V_u}{b_0} + \frac{\gamma_v M_u c_{AB}}{J_c}$$

where

γ_v is the factor used to determine the fraction of unbalanced factored moment, M_u transferred by eccentricity of shear at the centroidal axis c-c of the critical section.

In accordance with the EFM solution, the transfer of moment from one principal direction at a time is to be considered and presented in this example. The transfer of moment from the orthogonal directions using consistent load cases and combinations needs to be accounted for separately. For a typical corner column, the orthogonal effects of the moment transfer would be quite significant.

The factored shear force, $V_u = V_{u,c}$ for the interior and exterior columns

The factored self-weight and any factored superimposed surface dead and live load acting within the critical section can be subtracted from the factored reaction at the slab-column joint, $V_{u,c}$. This procedure is applied in spSlab Program and is more accurate. In this example, we will proceed with more conservative approach where $V_u = V_{u,c}$.

The factored unbalanced moment, $M_u = M_{sc} + V_{u.c} \times \left[c_1 / 2 - \left(c_{AB} - d / 2 \right) \right]$ for the exterior column

The factored unbalanced moment, $M_u = M_{sc}$ for the interior column

The factored self-weight and any factored superimposed surface dead and live load acting within the critical section multiplied by moment arm, $b_1 / 2 - \left(c_{AB} - d / 2 \right)$ can be subtracted from the factored unbalanced moment, M_u.

For simplicity, spSlab Program utilizes $c_1/2-(c_{AB}-d/2)$ as the moment arm in M_u calculations. In this example, we will proceed with more conservative approach where M_u is calculated as shown above.

Without shear reinforcement in the slab, the equivalent concrete stress corresponding to nominal two-way shear strength of slab, v_n , equals to the stress corresponding to nominal two-way shear strength provided by concrete, v_c .

$$v_n = v_c = \min\left[4\lambda\sqrt{f'_c}\ ,\ \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f'_c}\ ,\ \left(\frac{\alpha_s d}{b_0}+2\right)\lambda\sqrt{f'_c}\right]$$

β = the ratio of the long to the short side of the supporting column

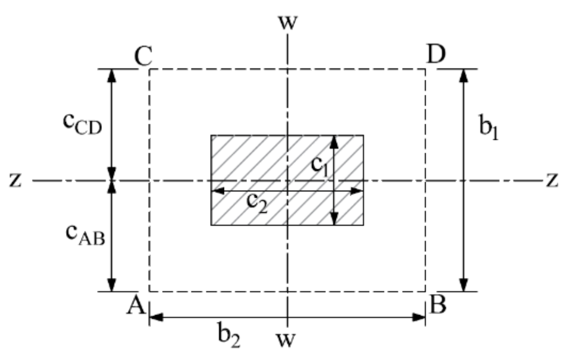
α_s = a constant dependent on supporting column location

λ = 1.0 (normal weight concrete)

b_0 = the perimeter of the critical section for two-way shear. The critical section shall be located so that the perimeter, b_0, is a minimum but need not be closer than d/2 to the perimeter of the supporting column per ACI 318-14, 22.6.4.1.

Interior column:

Determine the section properties for shear stress computations.



Critical shear perimeter for interior column

Critical Shear Perimeter for Interior Column

The location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_1}{2} = \frac{21.75}{2} = 10.88 \text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + 2 b_2 d c_{AB}^2$$

$$J_c = 2\left(\frac{21.75\times5.75^3}{12} + \frac{5.75\times21.75^3}{12} + (21.75\times5.75)(0)^2\right) + 2\times21.75\times5.75\times5.99^2 = 40131\ \text{in.}^4$$

The direct factored shear force, V_u = 75.91 kip

The factored unbalanced moment at the centroid of the critical section, M_u = 11.52 kips-ft

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_0 \times d} + \frac{\gamma_v M_u c_{AB}}{J_c}$$

$$v_u = \frac{75.91\times1000}{87\times5.75} + \frac{0.400\times11.52\times12\times1000\times10.88}{40131}$$

$$v_u = 151.7 + 15.0 = 166.7\ \text{psi}$$

The two-way design shear strength without shear reinforcement, v_r, can be calculated as:

$$v_c = \min\left[4\lambda\sqrt{f'_c}\ ,\ \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f'_c}\ ,\ \left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f'_c}\right]$$

$$v_c = \min\left[4\times1\times\sqrt{4000}\ ,\ \left(2+\frac{4}{1}\right)\times1\times\sqrt{4000}\ ,\ \left(\frac{40\times5.75}{87}+2\right)\times1\times\sqrt{4000}\right]$$

$$v_c = \min[253\ ,\ 279.5\ ,\ 293.7]\ \text{psi} = 253\ \text{psi}$$

$$\phi v_n = \phi v_c = 0.75\times253 = 189.7\ \text{psi}$$

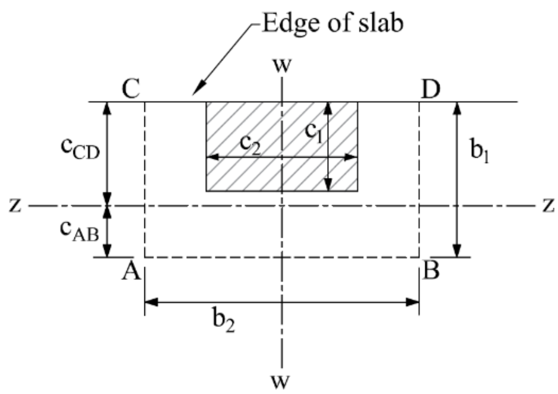
$$v_u / \phi v_n = 166.7/189.7 = 0.88 < 1.00\ \text{O.K.}$$

Since $\phi v_n \geq v_u$ at the critical section, the slab has **adequate** two-way (punching) shear strength at the interior column.

Note that, it may be plausible to assume that the effect of moment transfer in the orthogonal direction would result in similar magnitude of the two-way shear stress as in this direction which is 15.0 psi.

Exterior column:

Determine the section properties for shear stress computations.



Critical Shear Perimeter for Exterior Column

The location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{2(18.8 \times 5.75 \times 18.8/2)}{2 \times 18.8 \times 5.75 + 21.75 \times 5.75} = 5.99 \text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 d c_{AB}^2$$

$$J_c = 2\left(\frac{18.88 \times 5.75^3}{12} + \frac{5.75 \times 18.88^3}{12} + (18.88 \times 5.75)\left(\frac{18.88}{2} - 5.99\right)^2\right) + 21.75 \times 5.75 \times 5.99^2 = 14110 \text{ in.}^4$$

The direct factored shear force, V_u = 36.03 kip

The factored unbalanced moment at the centroid of the critical section,

$$M_u = 69.32 - 36.03 \times \left[8 - \left(5.99 - \frac{5.75}{2}\right)\right] \times \left(\frac{1}{12}\right) = 54.65 \text{ kip-ft}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_0 \times d} + \frac{\gamma_v M_u c_{AB}}{J_c}$$

$$v_u = \frac{36.03 \times 1000}{59.5 \times 5.75} + \frac{0.383 \times (54.65 \times 12 \times 1000) \times 5.99}{14110}$$

$$v_u = 105.3 + 106.6 = 211.9 \text{ psi}$$

The two-way design shear strength without shear reinforcement, ϕv_n, can be calculated as:

$$v_c = \min\left[4\lambda\sqrt{f'_c}\ ,\ \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f'_c}\ ,\ \left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f'_c}\right]$$

$$v_c = \min\left[4 \times 1 \times \sqrt{4000}\ ,\ \left(2+\frac{4}{1}\right) \times 1 \times \sqrt{4000}\ ,\ \left(\frac{40 \times 5.75}{87}+2\right) \times 1 \times \sqrt{4000}\right]$$

$$v_c = \min[253\ ,\ 279.5\ ,\ 293.7] \text{ psi} = 253 \text{ psi}$$

$$\phi\, v_n = \phi\, v_c = 0.75 \times 253 = 189.7 \text{ psi}$$

$$v_u / \phi v_n = 211.9 / 189.7 = 1.12 > 1.00 \text{ N.G.}$$

Since $\phi v_n < v_u$ at the critical section, the slab has **inadequate** two-way shear strength at the exterior column.

However, the two-way shear stresses at this exterior column can be reduced by modifying γ_f value per ACI 318-14, 8.4.2.3.4. The γ_f value may be increased up to 1.0, if the limitations on v_{ug} and ε_t in ACI 318-14, Table 8.4.2.3.4 for edge column where span direction is perpendicular to the edge (i.e. exterior column in this example) are satisfied. The increase in the γ_f value effectively decreases the γ_v value which, in turn, helps reduce the two-way shear stresses in lieu of the other costly alternatives such as increasing the slab thickness, increasing column size, employing drop panels, employing column capitals, increasing the concrete strength, f'_c, or providing shear reinforcement.

This procedure as defined in ACI 318-14 is provided as an option in spSlab Program and may be utilized in order to arrive at a satisfactory design.

For this exterior column, ACI 318-14, Table 8.4.2.3.4 permits the maximum modified value of γ_f be 1.0 if

$$v_{ug} \leq 0.75\phi v_c$$

where

v_{ug} = factored shear stress on the slab critical section for two-way action due to gravity loads without moment transfer.

and

$$\varepsilon_t \geq 0.004 \quad \text{within } b_{slab}$$

where

ε_t = net tensile strain in extreme layer of longitudinal tension reinforcement at nominal strength, excluding strains due to effective prestress, creep, shrinkage and temperature.

Determine v_{ug} as follows:

$$v_{ug} = \frac{V_u}{b_0 \times d} = \frac{36.03 \times 1000}{59.5 \times 5.75} = 105.3 \text{ psi} \leq 0.75\phi v_c = 0.75 \times 0.75 \times 253 = 142.3 \text{ psi.} \qquad \text{O.K.}$$

Determine the value of γ_v that is adequate for two-way shear design of the exterior column and the corresponding γ_f value as follows.

$$\frac{V_u}{b_0 \times d} + \frac{\gamma_v M_u c_{AB}}{J_c} = \phi v_c$$

$$\frac{36.03 \times 1000}{59.5 \times 5.75} + \frac{\gamma_v \times (54.65 \times 12 \times 1000) \times 5.99}{14110} = 189.7$$

$$105.3 + 278.4\gamma_v = 189.7$$

$$\gamma_v = \frac{189.7 - 105.3}{278.4} = 0.303$$

$$\gamma_f = 1 - \gamma_v = 1 - 0.303 = 0.697$$

Therefore, if the γ_f value is modified to 0.697, the two-way shear stress at this exterior column would be equal to two-way shear strength. Depending on the comfort level of the designer, $0.697 \leq \gamma_f \leq 1.0$ can be utilized per ACI 318-14, 8.4.2.3.4 for this column provided that the ε_t requirement within b_{slab} is met. In principle, a γ_f value closer to the lower-bound number (i.e. 0.697) would ensure better ductility while ensuring adequate two-way shear design. A γ_f value of 0.80 would provide approximately 15% cushion (i.e. $v_u \approx 85\,\%$ of ϕv_c) for two-way shear. In this example, the γ_f value of 1.0 will be utilized to compare with spSlab Program where an option to invoke the upper bound value of γ_f is provided.

$\gamma_f M_u = 1.0 \times 69.32$ kip-ft and $b_{slab} = 37$ in.

Assume $jd = 0.95 \times d = 0.95 \times 5.75 = 5.46$ in.

$$A_s = \frac{\gamma_f M_u}{\phi f_y jd} = \frac{1.0 \times 69.32 \times 12,000}{0.9 \times 60,000 \times 0.95 \times 5.75} = 2.82 \text{ in}^2$$

Try 15 - No. 4 bars. $A_s = 3.00 \text{ in}^2$

Compute the depth of equivalent rectangular stress block, a, for $A_s = 3.00 \text{ in}^2$; then recompute the A_s value using the computed value of the a:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3.00 \times 60,000}{0.85 \times 4,000 \times 37} = 1.43 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{1.43}{0.85} = 1.68 \text{ in.} < \frac{3 \times d}{8} = \frac{3 \times 5.75}{8} = 2.16 \text{ in.}$$

Therefore, the section is tension-controlled and capacity reduction factor, $\phi = 0.90$

$$A_s = \frac{\gamma_f M_u}{\phi f_y (d - a/2)} = \frac{1.0 \times 69.32 \times 12,000}{0.9 \times 60,000 \times (5.75 - 1.43/2)} = 3.06 \text{ in}^2$$

Try 16 - No. 4 bars. $A_s = 3.20 \text{ in}^2$

Compute a for $A_s = 3.20 \text{ in}^2$; then recompute A_s using that value of the a:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3.20 \times 60,000}{0.85 \times 4,000 \times 37} = 1.53 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{1.53}{0.85} = 1.80 \text{ in.} < \frac{3 \times d}{8} = \frac{3 \times 5.75}{8} = 2.16 \text{ in.}$$

Therefore, the section is tension-controlled and capacity reduction factor, $\phi = 0.90$

$$A_s = \frac{\gamma_f M_u}{\phi f_y (d - a/2)} = \frac{69.32 \times 12,000}{0.9 \times 60,000 \times (5.75 - 1.53/2)} = 3.09 \text{ in}^2 < A_{s,prov} = 3.20 \text{ in}^2 \qquad \text{O.K.}$$

The ε_t requirement needs to be checked before accepting this design.

Using a linear strain distribution,

$$\varepsilon_t = \varepsilon_s = \frac{d - c}{c} \times \varepsilon_{cu} = \frac{5.75 - 1.80}{1.80} \times 0.003 = 0.0066 > 0.004 \qquad \text{O.K.}$$

Therefore, modification of the γ_f value is permitted per ACI 318-14, 8.4.2.3.4.

The two-way shear stress (v_u) is:

$$v_u = \frac{V_u}{b_0 \times d} + \frac{\gamma_v M_u c_{AB}}{J_c}$$

$$v_u = \frac{36.03 \times 1000}{59.5 \times 5.75} + \frac{0.0 \times (54.65 \times 12 \times 1000) \times 5.99}{14110}$$

$$v_u = 105.3 + 0.0 = 105.3 \text{ psi} \leq \phi v_n = 189.7 \text{ ksi} \qquad \text{O.K.}$$

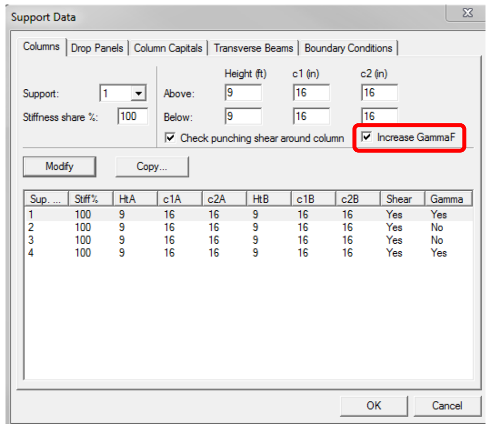
In flexural reinforcement calculations for unbalanced moment transfer by flexure, the utilization of an upper bound limit of 1.0 for γ_f results in 16 – No. 4 bars that need to be placed within b_{slab} width of 37 in. (i.e. upper limit of reliance of flexural reinforcement). On the other hand, in the two-way shear calculations, this results in a significant reserve capacity as the two-way shear stress is comprised of the direct shear only.

$$v_u / \phi v_n = 105.3 / 189.7 = 0.56 << 1.00$$

This presents an opportunity to reevaluate the slab thickness and look for more economical option. Alternatively, a lower value of can be utilized to reduce additional flexural reinforcement and maximize the punching shear compared with the available allowable shear strength of the slab.

Computer Program Solution

spSlab program provides an option to increase γ_f value (to 1.0 in this exterior column case). The spSlab output with regards to the hand solution above is provided below:



Increase γ_f Option within spSlab

Design Results - Flexural Transfer of Negative Unbalanced Moment at Supports

NOTE: *G - Increased GammaF factor.

Support	Width	Width-c	d	Munb	Comb	Patt	γf	As,req	As,prov	Add Bars	
	in	in	in	k-ft				in^2	in^2		
1	37.00	37.00	5.75	69.32	U1	All	1.000	3.070	0.881	11-#4	*G
2	37.00	37.00	5.75	11.52	U1	All	0.600	0.270	1.410	---	
3	37.00	37.00	5.75	11.52	U1	All	0.600	0.270	1.410	---	
4	37.00	37.00	5.75	69.32	U1	All	1.000	3.070	0.881	11-#4	*G

Design Results – Flexural Transfer of Negative Unbalanced Moment at Supports from spSlab

Design Results - Punching Shear Around Columns - Punching Shear Results

NOTE: *G - Decreased GammaV factor.

Support	Vu	vu	Munb	Comb	Patt	γv	vu	ΦVc	
	kip	psi	k-ft				psi	psi	
1	35.21	102.9	54.98	U1	All	0.000	102.9	189.7	*G
2	74.96	149.8	-11.52	U1	All	0.400	164.8	189.7	
3	74.96	149.8	11.52	U1	All	0.400	164.8	189.7	
4	35.21	102.9	-54.98	U1	All	0.000	102.9	189.7	*G

Design Results – Punching Shear Around Columns – Punching Shear Results from spSlab

Summary and Comparison of Results

The comparison of the results of Hand and spSlab solutions for moment transfer between slab and column by flexure with modified γ_f values at the exterior column is tabulated below and the results are in good agreement.

Comparison of Reinforcement Required within the Effective Slab Width, b_{slab}, for Moment Transfer between Slab and Column from Hand Solution and spSlab Solution		
Support Location	Exterior Column	
Solution Type	Hand	spSlab
Factored Slab Moment that is resisted by the Column at a joint, M_{sc} (ft-kips)	69.32	69.32
The Factor used to determine the Fraction of M_{sc} Transferred by Slab Flexure at Slab-Column Connections, γ_f	1.0	1.0
The Fraction of M_{sc} Transferred by Slab Flexure, γ_f M_{sc}, (ft-kips)	69.32	69.32
Effective slab width, b_{slab} (in.)	37	37
d (in.)	5.75	5.75
Reinforcement Required to Resist γ_f M_{sc} within b_{slab} (in^2)	3.09	3.07

Conclusions and Observations

The modification of γ_f value as permitted by ACI 318 code enables a satisfactory design to replace costly alternatives to limit punching shear impact on concrete floor thickness.

- γ_f is an important and useful tool that can be utilized to manage punching shear evaluation in lieu of costly options such as:
    - Increasing slab thickness
    - Increasing column size
    - Using drop panels
    - Using column capitals
    - Increasing slab f'_c
    - Adding shear studs
- γ_f offers the flexibility to transfer unbalanced moment by optimizing the proportion by which resistance is provided by the combination of shear and flexure.
- spSlab provides an upper-bound γ_f value for the user to evaluate punching shear options.